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On $\pi - \pi$ Correlations in Polarized Quark Fragmentation Using the Linear Sigma Model

John C. Collins

Department of Physics
Penn State University
104 Davey Laboratory
University Park, PA 16802, U.S.A.

and

Glenn A. Ladinsky

Department of Physics and Astronomy
Physics–Astronomy Building
Michigan State University
East Lansing, MI 48824, U.S.A.

ABSTRACT

Using the linear sigma model to describe quark–pion interactions, we compute polarization asymmetries in quark fragmentation. We show that the effects of transverse quark polarizations appear in the correlation between the two leading pions in a jet produced by the fragmentation of a quark. Such asymmetries provide a window to the nature of chiral symmetry breaking in QCD.

1. Introduction

Measuring the polarization of final state particles has been of continued interest in high energy particle physics. Such observations facilitate the measurement of the form factors which govern particle interactions and provide a probe of the symmetry breaking mechanisms present in nature.[1,2,3] One interesting idea is to probe the polarization carried by a quark or gluon of QCD through the distribution of its fragmentation products in a final state jet, first proposed by Nachtmann.[4,5] In particular, Nachtmann[4] showed that a three particle correlation in a jet can probe the helicity of the parton initiating the jet. This idea was rediscovered by Efremov, Mankiewicz and Törnqvist[6], and they named the correlation the 'handedness' of a jet.

In collaboration with Heppelmann[7], we showed that, in a similar fashion, a *two-particle* correlation can be used to probe the *transverse* polarization of the quark initiating a jet. This is a novel idea, and there is at present no known experimental information on this two-particle correlation. In this paper, we will perform some very simple model calculations of the transverse spin dependence of fragmentation. We will use a linear sigma model of pions coupled to quarks, somewhat in the spirit of Georgi and Manohar.[9] We will assume that this model, taken in the lowest relevant order of perturbation theory, is an approximation to the long distance dynamics of QCD. Within the model, we will calculate the spin-dependent part of the fragmentation of quarks to a two-pion state, $q \to \pi\pi + X$. The spin dependence shows up as a correlation between the plane of the two-pion system and the transverse spin vector of the quark.

The importance of our calculation is that it shows that there is consistency between the symmetries of QCD and a nonzero transverse-spin-dependence for fragmentation at the leading-twist level. Indeed, the large effect we calculate shows that there is no suppression of the spin-dependence. The spontaneous breaking of chiral symmetry (and hence a non-zero pion mass) is essential to our calculation. Moreover some kind of nontrivial phase or interference is also essential. Since we are in a strong-coupling regime, this does not preclude a large effect. Indeed, the size of the effect is very large—50% or more. In our simple model, the interference is between two-pion production in the continuum and at the σ resonance.

After reviewing the sigma model and the methods for computing decay functions, unpolarized quark fragmentation to a σ is calculated. This is used to normalize the polarization asymmetry. We then compute the polarized decay function to a two-pion state, and establish a nonvanishing asymmetry. This then demonstrates how the correlation between the two pions reflects the spin of the quark.

Finally we present some numerical calculations.

2. Sigma Model, Feynman Rules and Decay Functions

2.1. Sigma Model Lagrangian

We modify the sigma model lagrangian[10] to use quarks instead of nucleons:

$$\mathcal{L} = \frac{1}{2} \left[(\partial_\mu \sigma)^2 + (\partial_\mu \vec{\pi})^2 \right] + i\bar{q}\gamma^\mu \partial_\mu q - \tilde{g}\bar{q} \left(\sigma + i\vec{\tau} \cdot \vec{\pi}\gamma_5 \right) q - V \left(\sigma^2 + \vec{\pi}^2 \right) \qquad (2.1)$$

where

$$V = -\frac{\mu^2}{2} \left(\sigma^2 + \vec{\pi}^2 \right) + \frac{\lambda}{4} \left(\sigma^2 + \vec{\pi}^2 \right)^2 .$$

The Lagrangian possesses an $SU_R(2) \times SU_L(2)$ symmetry, and the term linear in the σ field that is normally used to provide explicit breaking of the chiral symmetry is not relevant for our purposes.

The sigma field is an isosinglet while the pion field is an isovector whose components may be written

$$\vec{\pi} = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix} . \qquad (2.2)$$

By using the isospin invariance of the pion field we can define the charged and neutral pion fields,

$$\pi^\pm \equiv \frac{1}{\sqrt{2}} \left(\pi_1 \mp i\pi_2 \right), \qquad \pi^0 \equiv \pi_3. \qquad (2.3)$$

The quark field is an isodoublet with three colors,

$$q = \begin{pmatrix} u \\ d \end{pmatrix} . \qquad (2.4)$$

The $(\tau^a)_{ij}$ are the 2×2 Pauli matrices,

$$\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2.5}$$

We use an index $a = 1, 2, 3$ for the pion fields and indices $i, j = 1, 2$ for the quark fields.

Because of the wrong sign mass term in Eq. (2.1), there is spontaneous symmetry breaking. We choose the σ field to have the nonzero vacuum expectation value:

$$\langle 0|\sigma|0\rangle = v, \qquad \langle 0|\vec{\pi}|0\rangle = 0, \qquad v = \sqrt{\mu^2/\lambda}. \tag{2.6}$$

After the transformation $\sigma \to \sigma + v$, the quark interaction becomes

$$-\tilde{g}\bar{q}\left(\sigma + i\vec{\tau} \cdot \vec{\pi}\gamma_5\right)q \longrightarrow -\tilde{g}v\bar{q}q - \tilde{g}\bar{q}\left(\sigma + i\vec{\tau} \cdot \vec{\pi}\gamma_5\right)q, \tag{2.7}$$

and the remaining $\vec{\pi}$ and σ interactions become

$$-V\left(\sigma^2 + \vec{\pi}^2\right) \longrightarrow -\mu^2\sigma^2 - \lambda v\sigma^3 - \frac{\lambda}{4}\sigma^4 - \frac{\lambda}{4}\vec{\pi}^4 - \frac{\lambda}{2}\sigma^2\vec{\pi}^2 - \lambda v\sigma\vec{\pi}^2. \tag{2.8}$$

Note that the pion field is massless, the σ has a mass of $\sqrt{2}\mu$ and the quarks have gained a mass of $\tilde{g}v$. If we take the up quark mass to be around $300\,\mathrm{MeV}$ and the vacuum expectation value of the sigma field for the spontaneous symmetry breaking to be $v = f_\pi = 92\,\mathrm{MeV}$, then $\tilde{g} \approx 3$. Typically, the sigma mass is taken to be around[11] $600\,\mathrm{MeV}$. The couplings $\tilde{g}$ and λ are large, as is appropriate for hadronic interactions in the non-perturbative regime.

2.2. Feynman Rules and Decay Functions

Among the Feynman rules obtained from the Sigma Model lagrangian after spontaneous symmetry breaking are the vertex factors, as displayed in Fig. 2.1. To perform a calculation involving charged pions it is useful to introduce isospin "polarization" vectors:

$$\varepsilon_{\mp}^a = (1, \pm i, 0)/\sqrt{2}, \qquad \varepsilon_0^a = (0, 0, 1). \tag{2.9}$$

An incoming state gets an $\vec{\varepsilon}$ and an outgoing state gets an $\vec{\varepsilon}^*$. A color average over the initial state is used.

In our model, we will compute an approximation to the spin-dependent fragmentation of a quark with helicity h and space-like transverse spin vector s_T^μ. The total spin will
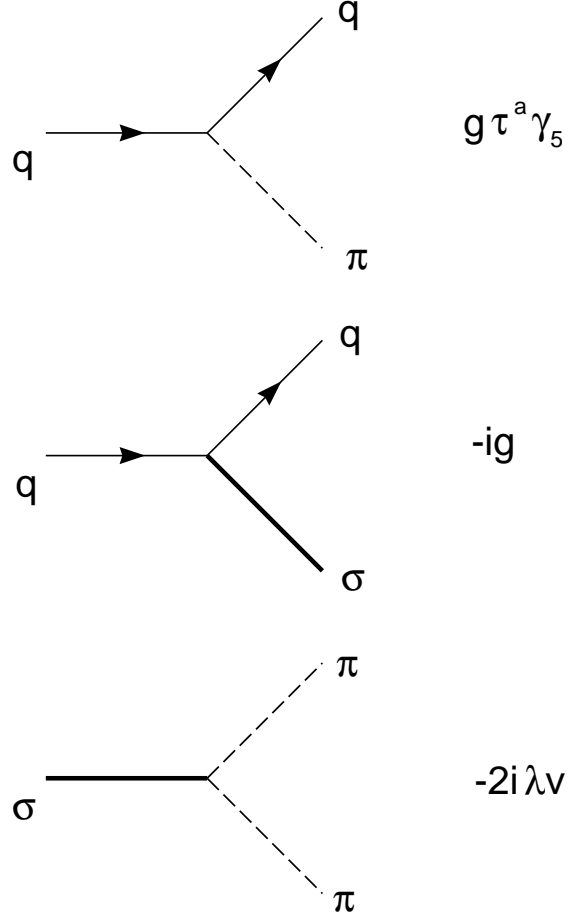
Figure 2.1: The relevant Feynman rules obtained from the Sigma Model lagrangian.

obey the constraint $0 \leq h^2 - s_T^2 \leq 1$. (Note that with the usual metric, $s_T^2 \leq 0$.) In the definition of the fragmentation functions[7,8] there is a trace with the following Dirac matrices:

$$G = \frac{1}{2} \left(\gamma^+ + h\gamma^+\gamma_5 + \gamma^+\gamma_5 \; \slashed{s}_T \right). \tag{2.10}$$

The light-cone component, manifest in the overall factor of γ^+, provides the projection on the leading twist part of the fragmentation. Eq. (2.10) gives the Feynman rule for the cut eikonal vertex that defines the fragmentation function. The γ^+ term gives the unpolarized fragmentation. The helicity dependent term $h\gamma^+\gamma_5$ will not contribute since we only measure two particles in the final state; a helicity dependent asymmetry only occurs with the three-particle correlation called "handedness".[4,5,6]
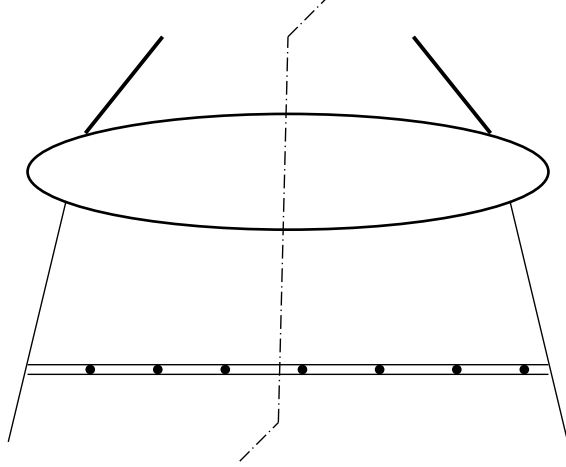
Figure 2.2: The fragmentation of a ψ into hadron A.

The Feynman rules for fragmentation functions were written down in Ref. 12, with the polarization dependence given in Ref. 7Ref. 8. The distribution of a single measured hadron A of momentum p in the fragmentation of a quark is a function of the longitudinal momentum fraction z of the hadron and of the transverse momentum $\mathbf{k}_T$ of the quark relative to the hadron. It has the form[8]

$$\hat{D}_{A/\psi}(z, \mathbf{k}_T) = \int \frac{dk^-}{(2\pi)^4} \Phi_{A/\psi}(k^2, k \cdot p), \tag{2.11}$$

where $k^+ = p^+/z$, and $\Phi_{A/\psi}(k^2, k \cdot p)$ is the inclusive two–point function,

$$\Phi_{A/\psi}(k^2, p \cdot k) = \frac{1}{6} \int d^4 x e^{ik \cdot x} \operatorname{Tr} G\langle 0| \psi(x) a^{\dagger}_{\mathrm{out}A}(p) a_{\mathrm{out}A}(p) \bar{\psi}(0) |0\rangle, \tag{2.12}$$

with the Dirac matrix G being defined in Eq. (2.10). The overall factor $1/6$ represents an average over quark color and spin. This is represented diagrammatically in Fig. 2.2. The factor $a^{\dagger}_{\mathrm{out}A}(p) a_{\mathrm{out}A}(p)$ is the number operator which counts how many hadrons of type A and momentum p are in a state. Note that the transverse momentum of the hadron relative to the quark is $-z\mathbf{k}_T$.

The usual decay function is obtained by integrating over transverse momentum:

$$D_{A/\psi}(z) = \int d^2 \mathbf{k}_T \hat{D}_{A/\psi}(z, \mathbf{k}_T). \tag{2.13}$$

It has no dependence on the quark spin (h, s). To interpret the fragmentation function as a probability would require an extra factor, but since we will be computing polarization asymmetries, this factor will cancel between the numerator and the denominator, so we will ignore it. For further details the reader is referred to Ref. 12 and Ref. 8.

Exactly similar formulae apply to fragmentation to 2 measured particles. We simply replace the number operator $a^\dagger_{\text{out}A}(p)a_{\text{out}A}(p)$ by the corresponding operator for two particles:

$$a^\dagger_{\text{out}A_1}\left(p_1\right) a^\dagger_{\text{out}A_2}\left(p_2\right) a_{\text{out}A_2}\left(p_2\right) a_{\text{out}A_1}\left(p_1\right),$$

so that the fragmentation function to two particles is defined by

$$
\begin{aligned}
\hat{D}_{H/a}\left(z, k_\perp\right) &\equiv \sum_X \int \frac{dy^- d^2 y_\perp}{12\left(2\pi\right)^3} e^{ik^+ y^- - ik_\perp \cdot y_\perp} \\
&\times \operatorname{Tr} \gamma^+ \langle 0|\psi_a\left(0, y^-, y_\perp\right)|A_1 A_2 X\rangle\langle A_1 A_2 X|\bar{\psi}_a\left(0\right)|0\rangle.
\end{aligned}
\tag{2.14}
$$

The polarization-dependent part is[7,8]

$$
\begin{aligned}
\Delta\hat{D}_{H/a}\left(z, k_\perp, s_\perp\right) &\equiv \sum_X \int \frac{dy^- d^2 y_\perp}{12\left(2\pi\right)^3} e^{ik^+ y^- - ik_\perp \cdot y_\perp} \\
&\times \operatorname{Tr} \gamma^+ \gamma_5 \gamma_\perp \cdot s_\perp \langle 0|\psi_a\left(0, y^-, y_\perp\right)|A_1 A_2 X\rangle\langle A_1 A_2 X|\bar{\psi}_a\left(0\right)|0\rangle.
\end{aligned}
\tag{2.15}
$$

Ultimately we will integrate over $k_\perp$, the transverse momentum of the quark relative to the measured hadrons, and in particular this will imply an azimuthal average over $k_\perp$. In this situation, to get transverse-spin dependence will require that there be an imaginary part in the amplitudes making up Eq. (2.15) (since the factor of γ_5 will give a factor i). In lowest order for the fragmentation $q^* \to \pi\pi X$, the imaginary part will come from the σ propagator, as in Fig. 2.3(c) and (d).

First, we will compute the fragmentation for $q^* \to \sigma X$, followed by the decay $\sigma \to \pi\pi$. This will give us the total rate for $q^* \to \pi\pi X$ at the resonance. Then we will compute the spin dependence from interference graphs for $q^* \to \pi\pi q$, with a correct treatment of the imaginary part of the denominator of the σ propagator

The momentum of the fragmenting quark is denoted by k and, depending upon the situation, the momentum of either the σ or $\pi\pi$ pair is represented by p. The Lorentz frame chosen is that where the sigma particle (or rather the combined momentum of the two pions of interest) has zero transverse momentum. The longitudinal momentum fraction of the
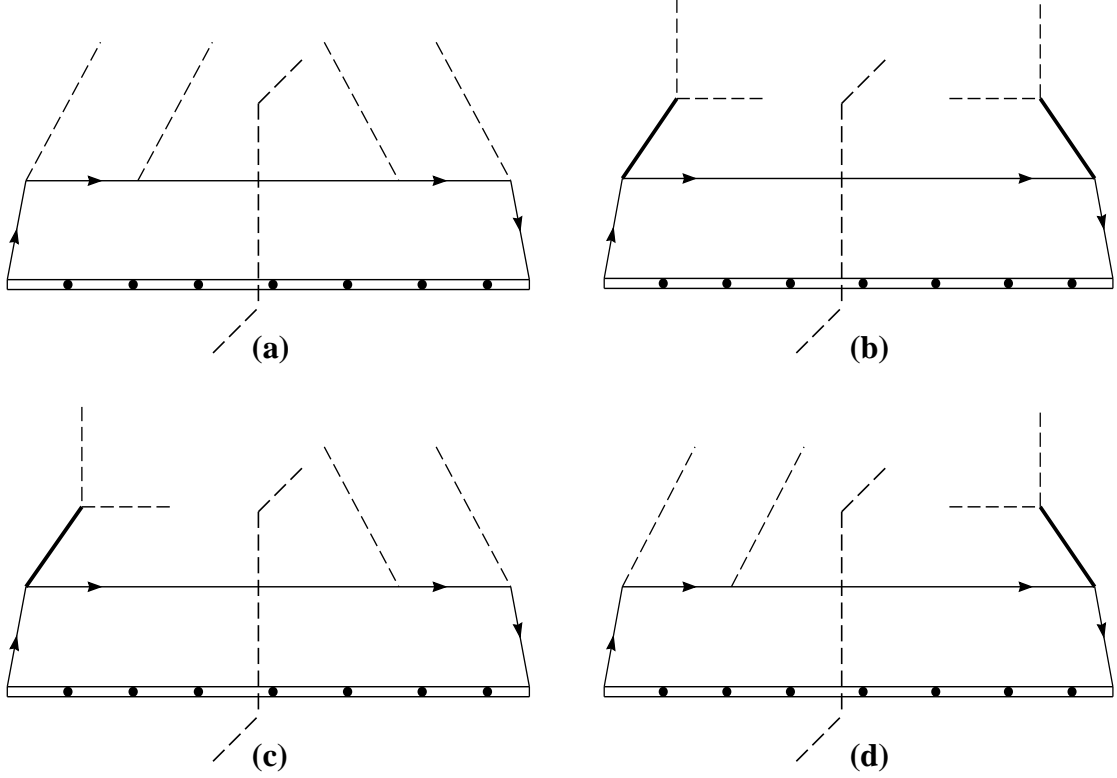
Figure 2.3: The four cut diagrams for the $u^* \to \pi^+ \pi^- u$ fragmentation

two-pion system (or σ) is given by $z \equiv p^+/k^+$. Generally, these two-pion system variables will be written in terms of individual pion variables,

$$z = z_1 + z_2, \qquad p = p_1 + p_2, \tag{2.16}$$

where the individual pion variables will be defined according to the process considered.

2.3. Decay Width of Sigma

The imaginary part that we need for the spin-dependence comes from the imaginary part of the denominator of the σ propagator. At lowest order, this will be given by the width of the σ. Recent work has appeared where the random phase approximation[13] and one–loop computations[14] have been applied to the linear sigma model to compute the width $\Gamma(\sigma \to \pi\pi)$. However, since we are simply demonstrating the existence of a spin observable in this paper, we shall not enter the quantitative detail of these references.

With the $\sigma - \pi - \pi$ coupling described by Fig. 2.1, the decay width may be expressed as

$$\Gamma\left(\sigma \rightarrow \pi\pi\right) = \frac{3\lambda^2 v^2}{8\pi m_\sigma}, \qquad (2.17)$$

where two thirds of the width is due to the decay to charged pions and one third of the width is due to the decay to the neutral pion pair. Using $m_\sigma = 600\,\mathrm{MeV}$, $m_q = 300\,\mathrm{MeV}$, and $v = 92\,\mathrm{MeV}$, we find that $\Gamma(\sigma \rightarrow \pi\pi) = 761\,\mathrm{MeV}$. The relations between the parameters which exist due to the spontaneous symmetry breaking, allow us to write the width in other forms:

$$\Gamma\left(\sigma \rightarrow \pi\pi\right) = \frac{3\tilde{g}^2 m_\sigma^3}{32\pi m_q^2} = \frac{3m_\sigma^3}{32\pi v^2}. \qquad (2.18)$$

3. Fragmentation $q^* \rightarrow \sigma X$

Our goal is to compute the $\pi\pi$ asymmetry near the σ pole, so we next find the total production rate for fragmentation to σ. This rate will provide the normalization for the polarization asymmetries. The distribution to be evaluated is represented by the diagram in Fig. 2.3(b), but with the pion loop removed. The momenta are defined such that k represents the incoming virtual quark momenta while p_3 and p respectively represent the outgoing quark and sigma momentum. (The momenta p_1, p_2 will represent the pion momenta in $q^* \rightarrow q\pi\pi$.) The $+$ momentum fraction carried away by the sigma is given by z. Summing over final state quark spins and averaging over initial state spins and colors yields the relation

$$\hat{D}_{\sigma/q}\left(z, k_T\right) = 2\pi\tilde{g}^2 \int \frac{dk^+ dk^-}{(2\pi)^4} \frac{\mathrm{Tr}\left[(\not{p}_3 + m_q)(\not{k} + m_q)\gamma^+(\not{k} + m_q)\right]}{4\left(k^2 - m_q^2\right)^2}$$
$$\delta\left(k^+ - p^+/z\right)\delta\left((k-p)^2 - m_q^2\right), \qquad (3.1)$$

where the computation is being done in $d = 4$ spacetime dimensions. There will be no spin dependence for a single particle fragmentation after we integrate over the transverse momentum (or at least its azimuth).

The definition of the fragmentation function is that we use a frame in which the σ has zero transverse momentum, $\mathbf{p}_T = 0$. A simple calculation yields

$$\hat{D}_{\sigma/q}\left(z, k_T\right) = \frac{\tilde{g}^2 z^2}{16\pi^3} \frac{m_q^2\left(2 - z\right)^2 + z^2 k_T^2}{\left[m_\sigma^2\left(1 - z\right) + m_q^2 z^2 + z^2 k_T^2\right]^2}. \qquad (3.2)$$

Normally, fragmentation functions are defined with an integral over k_T, the transverse momentum of the quark relative to the measured hadrons. These integrals have divergences at large k_T which are removed by renormalization (or some other means, like a cut off). The dependence on the cut-off or on the renormalization scale gives the usual Altarelli-Parisi evolution. For our purposes, however, it will be convenient not to bother with the integral. In any event, our model is only appropriate integrated over low k_T, where QCD is non-perturbative: The fragmentation functions would then be suitable initial data for normal perturbative QCD evolution.

Therefore we will not present the results integrated over k_T.

To obtain the resulting fragmentation function to $\pi^+\pi^-$, we must insert the decay $\sigma \to \pi^+\pi^-$ into the quark fragmentation derivation. This is given by the sigma propagators and the pion loop in Fig. 2.3(b), which result in Eq. (3.1) being multiplied by a factor

$$\left| \frac{i\,(-2i\lambda v)}{p^2 - m_\sigma^2 + im_\sigma\Gamma} \right|^2. \tag{3.3}$$

We are aware that the width of the sigma is in the neighborhood of its mass (using $m_\sigma = 600\,\mathrm{MeV}$, $m_q = 300\,\mathrm{MeV}$, and $v = 92\,\mathrm{MeV}$), but since our main interest is to demonstrate a nonzero asymmetry, and its order of magnitude, we just use the narrow width approximation,

$$\left| \frac{i}{p^2 - m_\sigma^2 + im_\sigma\Gamma} \right|^2 \approx \frac{\pi}{m_\sigma\Gamma}\delta\left(p^2 - m_\sigma^2\right). \tag{3.4}$$

For the total rate, and in the narrow width approximation, the graphs in Fig. 2.3 other than (b) are unimportant, since they are less singular at $p^2 = m_\sigma^2$. Hence the lowest order approximation for unpolarized fragmentation to $\pi^+\pi^-$ pairs is

$$\hat{D}_{\pi^+\pi^-/q} = \frac{32\pi^2}{3}\hat{D}_{\sigma/q}\left(z_1 + z_2, \mathbf{k}_T\right). \tag{3.5}$$

The factor $32\pi^2/3$ can be considered as $16\pi^2$ times the branching ratio, $2/3$, to $\pi^+\pi^-$.

4. Polarized Partons and Two–Particle Fragmentation

We now compute the dependence on transverse spin of the fragmentation $q \to \pi^+ \pi^- X$, using the definition Eq. (2.15). Since our asymmetry will be at the σ pole, we define the sigma momentum as the combined momentum of the two final state pions.

We consider the decay of an up quark which produces a charged pion pair. At lowest order, there are four Feynman diagrams for the $u^* \to \pi^+ \pi^- u$ fragmentation, and they are shown in Fig. 2.3. Topologically, this decay is the same as the decay for the down quark, $d^* \to \pi^+ \pi^- d$. Likewise, nonzero asymmetries are also expected from the fragmentation to $\pi^\pm - \pi^0$ pairs. The quark decays to $\pi^0 \pi^0$ pairs will produce a smaller asymmetry, because the azimuthal dependence is $\sin \phi$, and Bose symmetry of the two pions therefore requires a zero in the spin asymmetry at $z_1 = z_2$.

4.1. Getting the Polarization Dependence

All the spin dependences for the lowest order $u^* \to \pi^+ \pi^- u$ are in the imaginary parts of the traces from the cut diagrams. Since the decay functions must be real, combining the results from all of the relevant tree diagrams will result in the cancellation of all of the spin contributions for the total decay functions unless there is another factor present with an imaginary part. Such is the case when we are at a resonance.

Of the diagrams in Fig. 2.3 for $u^* \to \pi \pi u$, the only ones which can contribute a transverse spin dependence to the decay functions are (c) and (d). They have interference between the continuum and a σ propagator. We next note that these diagrams each have one factor of the σ-propagator, for which we can write

$$\frac{i}{p^2 - m_\sigma^2 + i m_\sigma \Gamma} = PV \frac{i}{p^2 - m_\sigma^2} + \pi \delta \left(p^2 - m_\sigma^2 \right), \tag{4.1}$$

in the narrow width approximation. The sum of the lowest order decay function amplitudes represented by Fig. 2.3(c) and (d) can then be expressed as

$$2\Re \left[\frac{a + ib}{p^2 - m_\sigma^2 - i m_\sigma \Gamma} \right]. \tag{4.2}$$

Here a and b are smooth functions of the kinematic variables in the vicinity of the σ pole. Moreover, as we will see, the spin dependence resides only in the b coefficient.

The principal value in Eq. (4.1) implies that the spin-independent term gives a small contribution when integrated over a neighborhood of the σ pole, so that Eq. (3.5) gives the unpolarized fragmentation. Nevertheless, it is Eq. (4.2) that gives the singular part of the spin dependence:

$$2\pi\delta\left((p_1+p_2)^2-m_\sigma^2\right)b. \tag{4.3}$$

The decay function can then be expressed as the sum of an unpolarized contribution and a spin asymmetric piece

$$\hat{D}_{q^*\to\pi\pi q}\left(z_1,z_2,s_T,\mathbf{k}_T\right)=\hat{D}_{q^*\to\pi\pi q}^{unpol}\left(z_1,z_2,\mathbf{k}_T\right)+\Delta\hat{D}_{q^*\to\pi\pi q}^{pol}\left(z_1,z_2,s_T,\mathbf{k}_T\right). \tag{4.4}$$

In the narrow width approximation, both the unpolarized and the polarized term have a factor $\delta((p_1+p_2)^2-m_\sigma^2)$, so that the ratio, which is the spin asymmetry, is well-defined:

$$\frac{\Delta\hat{D}_{q^*\to\pi\pi q}^{pol}\left(z_1,z_2,s_T,\mathbf{k}_T\right)}{\frac{32\pi^2}{3}\hat{D}_{\sigma/q}\left(z_1+z_2,\mathbf{k}_T\right)}. \tag{4.5}$$

Here, we used Eq. (3.5) for the denominator.

4.2. The Amplitude Computations

We now compute the necessary terms from the diagrams in Fig. 2.3 for the fragmentation of an up quark into two charged pions. We denote π^+ momentum by p_2 and the π^- momentum by p_1, and we use a label (a,b,c,d) to indicate the particular diagram in Fig. 2.3.

$$F_c=-2\pi\tilde{g}^3\lambda vC_c\frac{\text{Tr}\left[(\not{p}_3+m_q)(\not{k}+m_q)G(\not{k}+m_q)(\not{k}-\not{p}_2-m_q)\right]}{\left(k^2-m_q^2\right)^2\left[(k-p_2)^2-m_q^2\right]\left[(p_1+p_2)^2-m_\sigma^2+im_\sigma\Gamma\right]}$$
$$\delta\left((k-p_1-p_2)^2-m_q^2\right), \tag{4.6}$$

The C_c factor carries the isospin factors and for $u^*\to\pi^+\pi^-u$, $C_c=C_d=2$. Graph (a) has no singularity at the σ pole, so that we do not need to calculate it for our approximation. We have already obtained the contribution of graph (b), in Eq. (3.5). Notice that F_d is the complex conjugate of F_c, so that we do not need to calculate it explicitly.

Therefore, we obtain the spin dependent portion of the decay function from

$$\Delta\hat{D}\left(z_1,z_2,\mathbf{s}_T,\mathbf{k}_T\right)=\left[\int\frac{dk^-}{(2\pi)^4}(F_c+F_d)\right]_{(p_1+p_2)^2=m_\sigma^2}=\Re\left[2\int\frac{dk^-}{(2\pi)^4}F_c\right]_{(p_1+p_2)^2=m_\sigma^2}, \tag{4.7}$$

If we equate $F_c + F_d$ from Eq. (4.6) with Eq. (4.2), we find the coefficient b for $u^* \to \pi^+ \pi^- u$ takes the value

$$
b = \frac{-2\pi \lambda v \tilde{g}^3 C_c \delta \left(p_3^2 - m_q^2\right)}{2 \left(p_1 \cdot p_3\right) \left[k^2 - m_q^2\right]^2}
$$
$$
\left\{ 4 \left[\left(\frac{p^+}{z}\right) \varepsilon \left(p_1, p_3, s, p\right) - h m_q \varepsilon \left(p_1, p_3, p, +\right) + \left(s \cdot k\right) \varepsilon \left(p_1, p_3, k, +\right) \right] \right.
$$
$$
\left. - \frac{1}{2} \left(p^2 + 2p \cdot p_3\right) \varepsilon \left(p_1, p_3, s, +\right) \right\},
$$

$$(4.8)$$

where $p_3 = k - p_1 - p_2$ is the momentum of the on–shell final state quark, and $p = p_1 + p_2$. We define $\epsilon(p, q, r, s) \equiv \epsilon_{\kappa\lambda\mu\nu} p^\kappa q^\lambda r^\mu s^\nu$, with the Levi-Civita symbol $\epsilon_{\kappa\lambda\mu\nu}$ obeying the following relations in light cone coordinates (LCC),

$$
\varepsilon^{\mu\nu\pm\pm} = 0
$$
$$
\varepsilon^{\mu\nu\pm\mp} = \mp \varepsilon^{\mu\nu 03} \qquad \left(\text{using } \frac{1}{\sqrt{2}} \text{ normalization per LCC index}\right) \qquad (4.9)
$$
$$
\varepsilon \left(p, k, s, 0\right) = - \left(\vec{p} \times \vec{k}\right) \cdot \vec{s},
$$

where the convention is set by $\varepsilon_{0123} = +1$.

Since we will use the fragmentation function after integrating over $\mathbf{k}_T$, we will average over the azimuth of $\mathbf{k}_T$. After this average, the helicity term vanishes because the formulas no longer maintain a sufficient number of Lorentz vectors to keep the contraction with the Levi–Civita tensor asymmetric in its indices. We also apply the narrow width approximation provided by Eq. (4.1). We obtain

$$
\Delta\hat{D} = \delta \left(\left(p_1 + p_2\right)^2 - m_\sigma^2\right) \frac{\tilde{g}^3 \lambda v}{2\pi^2} \frac{\hat{\mathbf{z}} \cdot \left(\vec{\mathbf{s}}_T \times \vec{\mathbf{p}}_{1T}\right)}{2 p_{1T}^2} \frac{z_\sigma^2 \left(1 - z_\sigma\right)}{\left[\left(k_T^2 + m_q^2\right) z_\sigma^2 + m_\sigma^2 \left(1 - z_\sigma\right)\right]^2}
$$
$$
\left[m_\sigma^2 z_2 - \frac{\left(k_T^2 + m_q^2\right) \left(z_1^2 z_2 m_\sigma^2 - 2 z_1 z_\sigma^2 p_{1T}^2\right) + \left(1 - z_\sigma\right) \left[z_2 \left(1 - z_\sigma\right) m_\sigma^2 - 4 z_1 z_\sigma m_q^2\right] p_{1T}^2}{\sqrt{\left[\left(1 - z_\sigma\right)^2 p_{1T}^2 + z_1^2 \left(k_T^2 + m_q^2\right)\right]^2 - 4 z_1^2 \left(1 - z_\sigma\right)^2 p_{1T}^2 k_T^2}} \right],
$$

$$(4.10)$$

where $z_\sigma = z_1 + z_2$ and $\hat{\mathbf{z}} = (0, 0, 1)$ in rectangular coordinates.

The main demonstration of this paper is complete. Using the linear sigma model, we have achieved a nonvanishing asymmetry in the transverse polarization of a quark at leading twist. The nonvanishing component in Eq. (4.10) was maintained through the interference of the continuum production of $\pi\pi$ pairs with the sigma resonance. Furthermore, it is apparent that the existence of Eq. (4.10) is dependent upon a nonzero value

for the expectation value of the sigma field, i.e., it is the broken chiral symmetry which permits this asymmetry to exist. Consequently, this asymmetry can (theoretically) probe the chiral nature of QCD. In the limit that the bare quark mass is zero, nonvanishing polarization effects remain and without singularity as long as z is not at its endpoints.

The asymmetry at the sigma resonance may be estimated by comparing the spin dependence of $q^* \to q\pi\pi$ given by Eq. (4.10) with the unpolarized fragmentation given by Eq. (3.5). One immediate consequence is that the the correlation of the pions' direction with the transverse quark spin is given by the cross product $(\vec{s}_T \times \vec{p}_{1T})$. This is in accordance with the general theory.[7] So, as the pion momentum vector rotates about the quark (jet) axis, the asymmetry exhibits a sinusoidal rise and fall as it moves with respect to the transverse spin direction of the quark.

We have used the perturbative approximations, low order graphs and the narrow width approximation ($\Gamma \ll m_\sigma$), but these are not good approximations. Moreover, the use of both quark and hadron degrees of freedom as we have done is a crude model. So the following quantitative calculations must be only considered very rough estimates. Nevertheless, they do indicate that the spin-dependence is as large as it can be.

In Table 4.1, we list some values of the asymmetry: the ratio of Eq. (4.10) to Eq. (3.5). We have replaced the triple product $(\vec{s}_T \times \vec{p}_{1T})$ by $|p_{1T}|$, so that the numbers represent the amplitude of the sinusoidal dependence of the pion production on the azimuthal angle about the jet axis.

Notice that the numbers are large. One might worry that the asymmetries go above 100%, and that within a lowest order perturbative calculation (from the graphs of Fig. 2.3) this would correspond to a negative cross section. However, we are working in the neighborhood of the pole of the σ. Therefore, it was essential to use a dressed propagator for the σ lines, to get a suitable imaginary part and resonance width. This means that our model is not totally self-consistent: We have performed a selective resummation of graphs. We have further made a narrow width approximation on the σ pole: This is obviously far from perfect for physical values of the couplings of our model.

Our approximations are valid in the weak coupling limit, $\tilde{g}, \lambda \ll 1$, whence the asymmetry is of order $\tilde{g}\sqrt{\lambda}$, which is then much less than one. The true strong interactions are, of course, strong, and so obtaining an asymmetry above unity from our calculation is not

Table 4.1: The asymmetry of Eq. (4.5) is tabulated for various values of k_T, z_1 and z_2 using $s_T \times p_{1T} \to |s_T||p_{1T}|$. We use the values $m_\sigma = 600\,\mathrm{MeV}$, $m_q = 300\,\mathrm{MeV}$, $v = 92\,\mathrm{MeV}$.

THE TRANSVERSE SPIN ASYMMETRY	
$z_1 = 0.4$, $z_2 = 0.3$	
$z_\sigma k_T$ (MeV)	Asymmetry
0	1.31
200	1.21
400	0.89
600	0.58
1200	0.20
$z_1 = 0.2$, $z_2 = 0.15$	
$z_\sigma k_T$ (MeV)	Asymmetry
0	0.31
200	0.31
400	0.34
600	0.39
1200	0.54

impossible. All it indicates is that the effect we are calculating suffers from no particular suppression. Hence we can expect substantial analyzing power for quark transverse spin from measurements of the azimuthal dependence of pion pairs.

Note that we have used $z_\sigma k_T$, rather than k_T, as the transverse momentum variable, since this represents the transverse momentum of the pion pair relative to the jet.

When $z_\sigma k_T$ gets large, the asymmetry decreases. This is expected, since the quark is then far off-shell, and the graphs approach their values with zero quark mass. When the quark mass is zero, there is exact helicity conservation along the quark lines, and hence there is no transverse-spin asymmetry. It can be easily checked from Eq. (4.10) and Eq. (3.5) that our calculated asymmetry is proportional to $1/k_T^2$ at large k_T.

At small z_σ, the calculated asymmetry appears to get smaller, with a broader distribution in $z_\sigma k_T$. This may be a good prediction. But our model should not be reliable at small z. A reasonable prejudice is only that the spin dependence should decrease at small

z, since there one expects hadron production to be independent of the flavor and spin state of the initiating quark: The associated pomeron and gluon physics are not in our model.

5.   Conclusion

Using the Linear Sigma Model to describe the fragmentation of a polarized quark has demonstrated the existence of a nonvanishing asymmetry from which polarization information can be obtained. Although the model should only be considered to give crude qualitative information, the large asymmetries we calculate do show that the spin correlation can be completely unsuppressed.

This supports the value of doing experiments to measure the asymmetry experimentally. It can be used, for example, as a method of obtaining the transverse spin dependence of quarks in a transversely polarized proton.[7]

It is clearly important to find better models for the fragmentation that include spin effects. Obviously, real fragmentation of a quark results in many more than two pions. This can easily dilute the effect of the physics we have modeled. Nevertheless, the number of pion pairs is fairly low if we restrict our attention to large z and fairly low invariant mass.

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